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LETTER TO THE EDITOR

Brillouin scattering in a superconducting simple metal

M Manfredini†, P F Bortignon‡ and C E Bottani§

† Dipartimento di Ingegneria Nucleare del Politecnico di Milano, 20133 Milano, Italy
‡ Dipartimento di Fisica, Università degli Studi di Milano, 20133 Milano, Italy
§ cINFM, Unità di Ricerca Milano Politecnico, 20133 Milano, Italy

Abstract. We compute the width of the Rayleigh surface-wave peak in the Brillouin spectrum of a superconducting simple metal. The electron–phonon interaction is considered to be responsible for the attenuation. The decay constant presents a discontinuity at T_c and may have a second discontinuity below T_c depending on the phonon frequency.

We consider inelastic light scattering by acoustic waves in isotropic metals with a single flat surface; in this case there is a single surface-wave excitation named after Rayleigh [1].

Acoustic waves can scatter light by two distinct mechanisms. There is a surface-ripple mechanism [2–5] by which the incident light is reflected from the dynamic acoustic deformation of the sample surface. There is also a bulk elasto-optic mechanism [4–6] in which the coupling of incident and scattered light occurs through the acoustic modulation dielectric constant of the sample. In metals the first mechanism is dominant by two orders of magnitude [2, 4].

The present theory of light-scattering cross sections for surface ripples in solids does not involve studying the shape of the spectral peak associated with the Rayleigh wave because this does not depend on the opacity of the sample.

From the above cited work, the expression for the cross section is

$$\mathrm{d}^2\sigma/\mathrm{d}\Omega\,\mathrm{d}\omega_{\mathrm{S}} \propto \mathrm{Re}\left\{\omega v_{\mathrm{T}}^3 Q^{x2} q_{\mathrm{L}}^z / [4 v_{\mathrm{T}}^4 Q^{x2} q_{\mathrm{L}}^z q_{\mathrm{T}}^z + (\omega^2 - 2v_{\mathrm{T}}^2 Q^{x2})^2]\right\} \tag{1}$$

where Q^x and ω are respectively the component of the excitation wave vector parallel to the surface and its frequency; all other quantities in equation (1) refer to the elastic properties of the medium and are defined in [3].

The factor $\mathrm{Re}\{\ldots\}$ has a pole for $\omega = \omega_{\mathrm{R}}$, that is at the frequency of the Rayleigh surface wave. If this frequency is given an infinitesimal imaginary part, we can write

$$\mathrm{Re}\{\ldots\} = f(\sigma) v_{\mathrm{T}} Q^x \delta(\omega - \omega_{\mathrm{R}}) \tag{2}$$

where $f(\sigma)$ is π times the residue at the Rayleigh pole and is plotted in [3].

We take the finite lifetime of phonons into account. For an excitation whose amplitude is damped as $\exp(-\Gamma t)$, the inelastic scattering cross section has a Lorentzian frequency dependence

$$\mathrm{d}^2\sigma/\mathrm{d}\Omega\,\mathrm{d}\omega_{\mathrm{S}} = \{(\Gamma/\pi)/[(\omega_{\mathrm{S}} - \omega_{\mathrm{I}} \pm \omega)^2 + \Gamma^2]\}\mathrm{d}\sigma/\mathrm{d}\Omega \tag{3}$$

with a full width at half maximum (FWHM)

$$\Delta\omega = 2\Gamma. \tag{4}$$

Thus the lineshape of the Rayleigh wave is given by equation (2) in which the function $\delta(\omega - \omega_R)$ is substituted by

$$(\Gamma/\pi)/[(\omega - \omega_R)^2 + \Gamma^2]. \tag{5}$$

In what follows, we consider a harmonic crystal without defects so, the mechanism of attenuation is the interaction with the electrons, and we study how this interaction changes in the transition from the normal to the superconducting state. For a first calculation we analyse bulk phonons and then extend the result to surface excitations.

The first-order interaction Hamiltonian is

$$\begin{aligned} H_{ep} &= \sum_{kk'8\lambda} g_{kk'\lambda}\varphi_{k'-k,\lambda} C^{\dagger}_{k's} C_{ks} \\ \varphi_{q,\lambda} &= A_{q,\lambda} + A^{\dagger}_{-q,\lambda} \qquad k' = k + q + Q \end{aligned} \tag{6}$$

where Q is a reciprocal lattice vector; at this wavelength we can neglect Umklapp processes and consider $Q = 0$. We make the reasonable assumption that the matrix elements depend only on the difference $k' - k$ and are the same in both the normal and superconducting states.

By the use of the Bogoliubov–Valatin canonical transformation, the Hamiltonian (6) can be made to take the form

$$\begin{aligned} H_{ep} &= \sum_{kk's\lambda} g_{kk'\lambda}\varphi_{k'-k,\lambda}\{(u_{k'}\gamma^{\dagger}_{k'\uparrow} + v_{k'}\gamma_{-k'\downarrow})(u_k\gamma_{k\uparrow} + v_k\gamma^{\dagger}_{-k\downarrow})\} \\ &= \sum_{kk's\lambda} g_{kk'\lambda}\varphi_{k'-k,\lambda}\{u_{k'}u_k\gamma^{\dagger}_{k'\uparrow}\gamma_{k\uparrow} + v_{k'}v_k\gamma_{-k'\downarrow}\gamma^{\dagger}_{-k\downarrow} \\ &\quad + u_{k'}v_k\gamma^{\dagger}_{k'\uparrow}\gamma^{\dagger}_{-k\downarrow} + v_{k'}u_k\gamma_{-k'\downarrow}\gamma_{k\uparrow}\} \end{aligned} \tag{7}$$

where u_k and v_k are the BCS coefficients [7].

In equation (7) there are four terms (figure 1): $\gamma_2^{\dagger}\gamma_1$ and $\gamma_1^{\dagger}\gamma_2$ describe an interaction in which a quasiparticle is scattered from state 1 to state 2 and vice versa—we call it the A process; $\gamma_2^{\dagger}\gamma_1^{\dagger}$ and $\gamma_1\gamma_2$ respectively create and annihilate two quasiparticles—the B process. The B process is possible only if the energy of the phonon is larger than twice the energy gap, that is $\hbar\omega_{q\lambda} \geqslant 2\Delta$.

The matrix elements $\langle F|H_{ep}|I\rangle$ are

$$\begin{aligned} &(\mathrm{A.1}) \quad (u_{k_2}u_{k_1} - v_{k_2}v_{k_1})g_{k_1k_2\lambda_0} \qquad &(\mathrm{B.1}) \quad (u_{k_1}v_{k_2} + v_{k_1}u_{k_2})g_{k_1k_2\lambda_0} \\ &(\mathrm{A.2}) \quad (u_{k_2}u_{k_1} - v_{k_2}v_{k_1})g^{*}_{k_1k_2\lambda_0} \qquad &(\mathrm{B.2}) \quad (u_{k_1}v_{k_2} + v_{k_1}u_{k_2})g^{*}_{k_1k_2\lambda_0}. \end{aligned} \tag{8}$$

To compute the decay constant of a phonon in the superconducting state we use the Fermi golden rule summing over all possible quasiparticle states.

For the A process we obtain

$$\Gamma^{A}_{q\lambda} = \frac{2\pi}{\hbar}\sum_{s}\sum_{k}|g_{q\lambda}|^2(u_k u_{k+q} - v_k v_{k+q})^2\{f(k) - f(k+q)\}\delta(E_k - E_{k+q} + \hbar\omega_{q\lambda}) \tag{9}$$

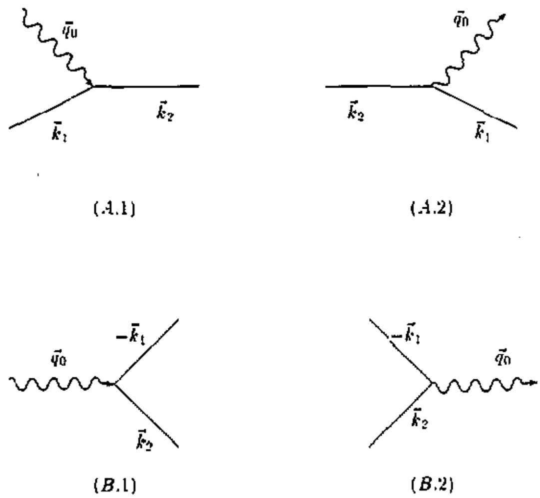

Figure 1. Graphical representation of inelastic scattering of phonons in a superconductor.

and for the B process

$$\Gamma^{\mathrm{B}}_{q\lambda} = \frac{2\pi}{\hbar}\sum_{s}\sum_{k}^{*}|g_{q\lambda}|^2(u_k v_{k+q} + v_k u_{k+q})^2\{1 - f(k) - f(k+q)\} \times \delta(\hbar\omega_{q\lambda} - E_k - E_{k+q}) \tag{10}$$

where

$$f(k) = f(E_k) = [\exp(E_k/k_{\mathrm{B}}T) + 1]^{-1}. \tag{11}$$

$\sum_k^*$ means that k is bounded on the upper part—in fact $E_k \leqslant \hbar\omega_{q\lambda} - E_{k+q}$; thus at maximum $E_k = \hbar\omega_{q\lambda} - \Delta$. Transforming the sums into integrals:

$$\Gamma^{\mathrm{A}}_{q\lambda} = (2V/\pi)|g_{q\lambda}|^2(m^{*2}/\hbar^5 q)I^{\mathrm{A}} \tag{12}$$

$$\Gamma^{\mathrm{B}}_{q\lambda} = (2V/\pi)|g_{q\lambda}|^2(m^{*2}/\hbar^5 q)I^{\mathrm{B}} \tag{13}$$

where

$$I^{\mathrm{A}} = \int_{\Delta}^{\infty} \mathrm{d}E \frac{E(E + \hbar\omega_{q\lambda}) - \Delta^2}{[E^2 - \Delta^2]^{1/2}[(E + \hbar\omega_{q\lambda})^2 - \Delta^2]^{1/2}}\{f(E) - f(E + \hbar\omega_{q\lambda})\} \tag{14}$$

$$I^{\mathrm{B}} = \int_{\Delta}^{\hbar\omega_{q\lambda} - \Delta} \mathrm{d}E \frac{E(\hbar\omega_{q\lambda} - E) + \Delta^2}{[E^2 - \Delta^2]^{1/2}[(\hbar\omega_{q\lambda} - E)^2 - \Delta^2]^{1/2}}\{1 - f(E) - f(\hbar\omega_{q\lambda} - E)\}. \tag{15}$$

Similar integrals have been reported in [8] for inelastic scattering of light by quasiparticles; however, the coherence factors obtained here for the phonon case are different from the ones for photons.

It is convenient to relate the phonon decay constant in the superconducting state to that in the normal metal at the same temperature. One can find the appropriate expression for the normal metal by the substitution $\Delta = 0$ in equation (12). The relative total decay constant becomes

$$\frac{\Gamma^{\mathrm{S}}}{\Gamma^{\mathrm{N}}} = \begin{cases} I^{\mathrm{A}} + I^{\mathrm{B}}/I^{\mathrm{N}} & \text{for } \omega \geqslant 2\Delta \\ I^{\mathrm{A}}/I^{\mathrm{N}} & \text{for } \omega < 2\Delta \end{cases} \tag{16}$$

where

$$I^{\mathrm{N}} = \int_0^\infty \mathrm{d}E\, \{f(E) - f(E + \hbar\omega_{q\lambda})\}. \tag{17}$$

Taking from BCS theory the temperature dependence of the gap parameter $\Delta(T)$, the integrals in equation (16) were solved numerically. A plot of $\Gamma^{\mathrm{S}}/\Gamma^{\mathrm{N}}$ against temperature for different phonon frequencies is shown in figure 2.

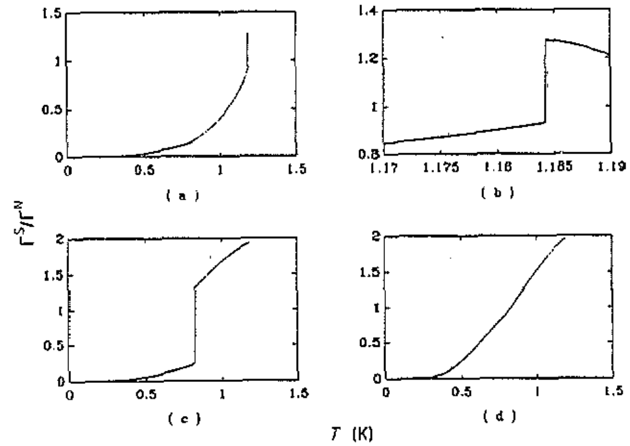

Figure 2. A plot of the relative phonon decay constant against temperature. The material is polycrystalline aluminium with $T_{\mathrm{c}} = 1.19$ K and $2\Delta(0) = 3.810^{-4}$ eV. Phonon frequencies: (a) and (b), $\hbar\omega = 4.55 \times 10^{-5}$ eV; (c), $\hbar\omega = 3.64 \times 10^{-4}$ eV; (d), $\hbar\omega = 4.55 \times 10^{-4}$ eV. Experimentally we can select these phonon frequencies by changing the wavelength of the incident laser beam.

The decay constant presents a discontinuity at T_{c}; a second discontinuity may appear below T_{c} depending on the phonon frequency. This is the consequence of the new channel of decay (B process) which is open only if $\hbar\omega \geqslant 2\Delta$.

Using the simple expression for the matrix elements of the electron–phonon interaction in the normal state given in [9] in the Thomas–Fermi approximation, we calculated the width of the Rayleigh peak in aluminium to be few hundreds of MHz. Moreover, we estimated the signal to noise ratio for a typical experimental set up in this Brillouin spectrum at a few Kelvin to be about twenty. Therefore, we may expect that a Brillouin scattering experiment in a superconductor at different temperatures should show the variation of the surface Rayleigh phonon lifetime as predicted in figure 2.

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